Multiscale modelling of neutron star oceans

BERN 18

Alice Harpole

Supervisors: Ian Hawke & Nils Andersson

Overview

- X-ray bursts & GR
- Modelling bursts
  - Relativistic detonations & deflagrations
  - Relativistic shallow water equations
  - Relativistic low Mach number approximation
  - Multiscale model
- Conclusions & future plans

What about GR?

- Strong gravity known to be important for NS physics
- NSs in LMXBs mostly fast rotators ($\gtrsim 200$ Hz)
- Coriolis force can drive spreading of flame front (Spitkovsky+ 02)
- Could *frame dragging* be important?
- Relativistic turbulence? (Radice & Rezzolla 13)

Modelling bursts

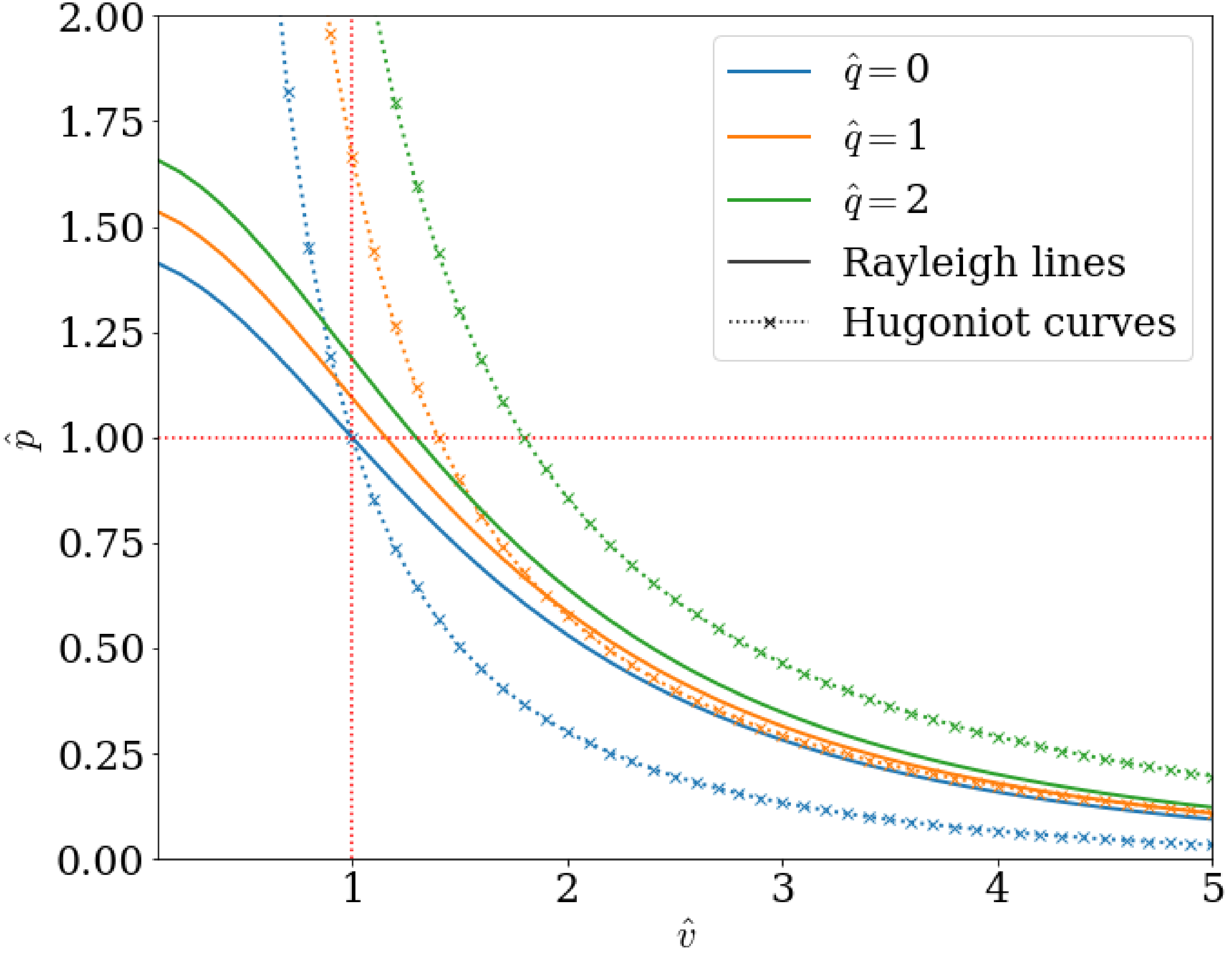
- We want to model **entire NS ocean** (or at least sizeable fraction of it)
- X-ray burst physics operates over **large range of scales**:
  - Coriolis force on large scales (~ 10 km)
  - burning reactions on small scales (~ 1 cm)
- Solution: use different models on different scales
- Shallow water $\rightarrow$ compressible $\rightarrow$ low Mach

Relativistic reactive RP

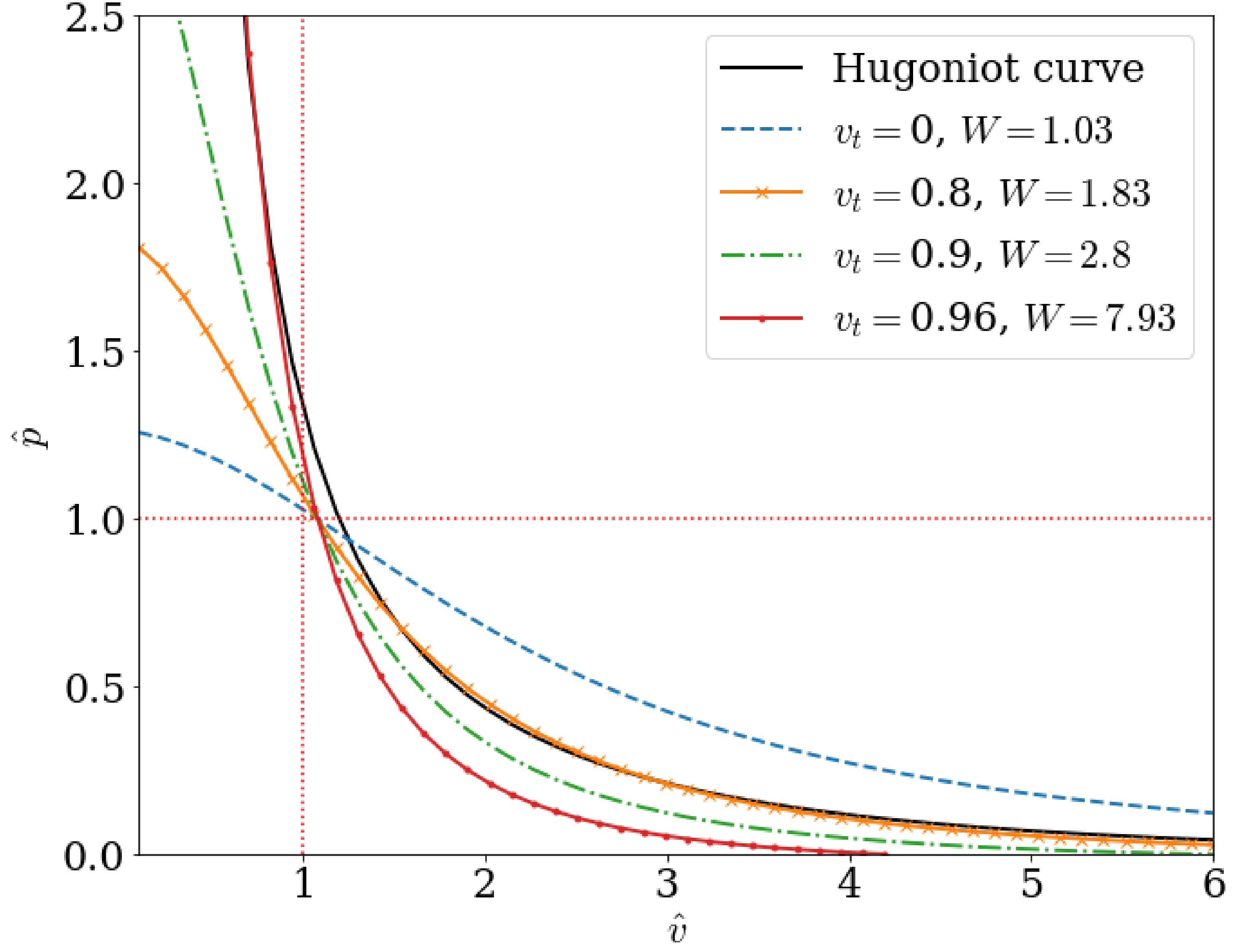
- On scale of whole NS, treat flame as discontinuity → can model as a Riemann problem
- In relativistic RP, Pons+ 00, Rezzolla & Zanotti 02 found fast tangential velocity can change solution
- What about reactive case? Is DDT possible (& relevant for NS oceans)?

Relativistic reactive RP

- Gao & Law 12 considered reactive relativistic RP
- Found presence of enthalpy in conservation of mass flux led to **both** Rayleigh & Hugoniot curves being functions of heat release q
- Varying v_t changes Rayleigh lines only
- Neither of these effects are present in Newtonian reactive RP

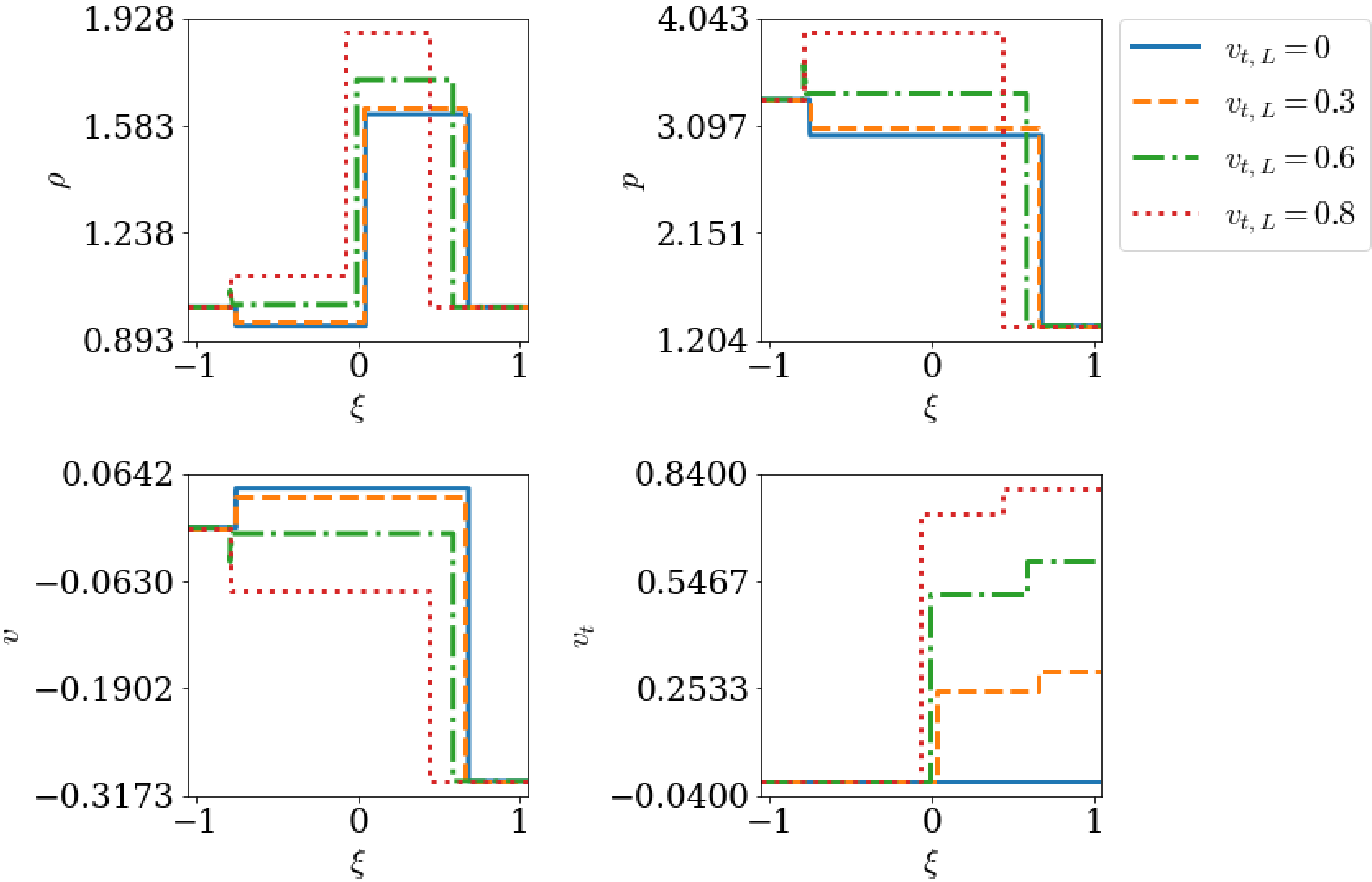


$\hat{q} = 0$
$\hat{q} = 1$
$\hat{q} = 2$
Rayleigh lines
Hugoniot curves
$\hat{p}$
$\hat{v}$
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
1
2
3
4
5


Hugoniot curve
$v_t = 0,\ W = 1.03$
$v_t = 0.8,\ W = 1.83$
$v_t = 0.9,\ W = 2.8$
$v_t = 0.96,\ W = 7.93$
$\hat{p}$
$\hat{v}$
2.5
2.0
1.5
1.0
0.5
0.0
1
2
3
4
5
6

Relativistic reactive RP

- `R3D2` is an exact solver for relativistic reactive RP
- Used to investigate problem further. We found:
  - Varying q and v_t **can** cause DDT
  - **But** require fine tuning of q to produce a system already on verge of transition or highly relativistic v_t
  - Therefore **probably not relevant** for astrophysical systems



$$(\rho, v_x, v_t, \varepsilon)_L = (1, 0, 0, 5.0), (\rho, v_x, v_t, \varepsilon)_R = (1, -0.3, v_t, 2), q_L = 0.01$$

$$(CJDF_{\leftarrow} R_{\leftarrow}) CS_{\rightarrow} \rightarrow WDF_{\leftarrow} CS_{\rightarrow} \rightarrow (CJDT_{\leftarrow} R_{\leftarrow}) CS_{\rightarrow} \rightarrow SDT_{\leftarrow} CS_{\rightarrow}$$

Relativistic shallow water

- On scale of entire NS, ocean layer is very thin ($\sim$10m vs $\sim$10km)
- **Shallow water equations** - integrate in vertical direction to reduce dimensionality of problem
- Used by Spitkovsky+ 02 to study effects of rotation on flame propagation
- In relativistic case, difficulties arise with defining vertical direction for non-static systems

Relativistic shallow water

- Newtonian 3-covariant (St-Cyr+ 08)

$$\frac{\partial u^i}{\partial t} + \gamma^{ij}\left[u^k \frac{\partial u_j}{\partial x^k} + \frac{\partial \Phi}{\partial x^j}\right] = 0,$$
$$\frac{\partial \Phi}{\partial t} + u^j \frac{\partial \Phi}{\partial x^j} + \frac{\Phi}{\sqrt{\gamma}} \frac{\partial}{\partial x^j}\left(\sqrt{\gamma} u^j\right) = 0$$

- GR 4-covariant (for static, spherically symmetric spacetime)

$$\nabla_\mu \left(\Phi u^\mu\right) = 0,$$
$$\nabla_\nu \left(\Phi u^\nu u_\mu\right) + \frac{1}{2} h^\nu_\mu \nabla_\nu \Phi^2 = 0$$

Multi-layer model

- Burning is confined to a **thin layer** of the ocean
- Model by splitting domain into multiple layers
- Add buoyancy, mass transfer and heating terms to shallow water equations to model layer interactions
- Problem is now '2.5d'

Low Mach

- Flame speed $\ll$ sound speed
- Motivates sound proof model
- We use **Low Mach Number** approximation:

$$M = \frac{v}{c_s} \ll 1$$

$$p(\vec{x}, r, t) \rightarrow p_0(r, t) + \pi(\vec{x}, r, t), \qquad \pi/p_0 = O(M^2)$$

- Previously used for Newtonian case (`MAESTRO`, Almgren+ 06) - we have extended to GR

Low Mach approximation

- Newtonian momentum equation

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \vec{\nabla} \vec{u} + \vec{\nabla} p = -\rho \, |g| \, \vec{e}_r$$

- Relativistic 3-covariant form

$$\left(\partial_t - \mathscr{L}_\beta\right) v_i + \alpha v^{k(3)} \nabla_k v_i + \frac{{}^{(3)}\nabla_i p}{H W^2} = \text{Source terms}$$

- Relativistic low Mach, $v_i = \bar{v}_i + V_i$

$$\left(\partial_t - \mathscr{L}_\beta\right) V_i + \alpha V^{k\,(3)} \nabla_k V_i + \frac{\beta_0}{H} {}^{(3)}\nabla_i \left(\frac{\pi}{\beta_0} \right) = -\frac{\varrho - \bar{\varrho}}{H} {}^{(3)}\nabla_i \alpha$$

Multiscale model

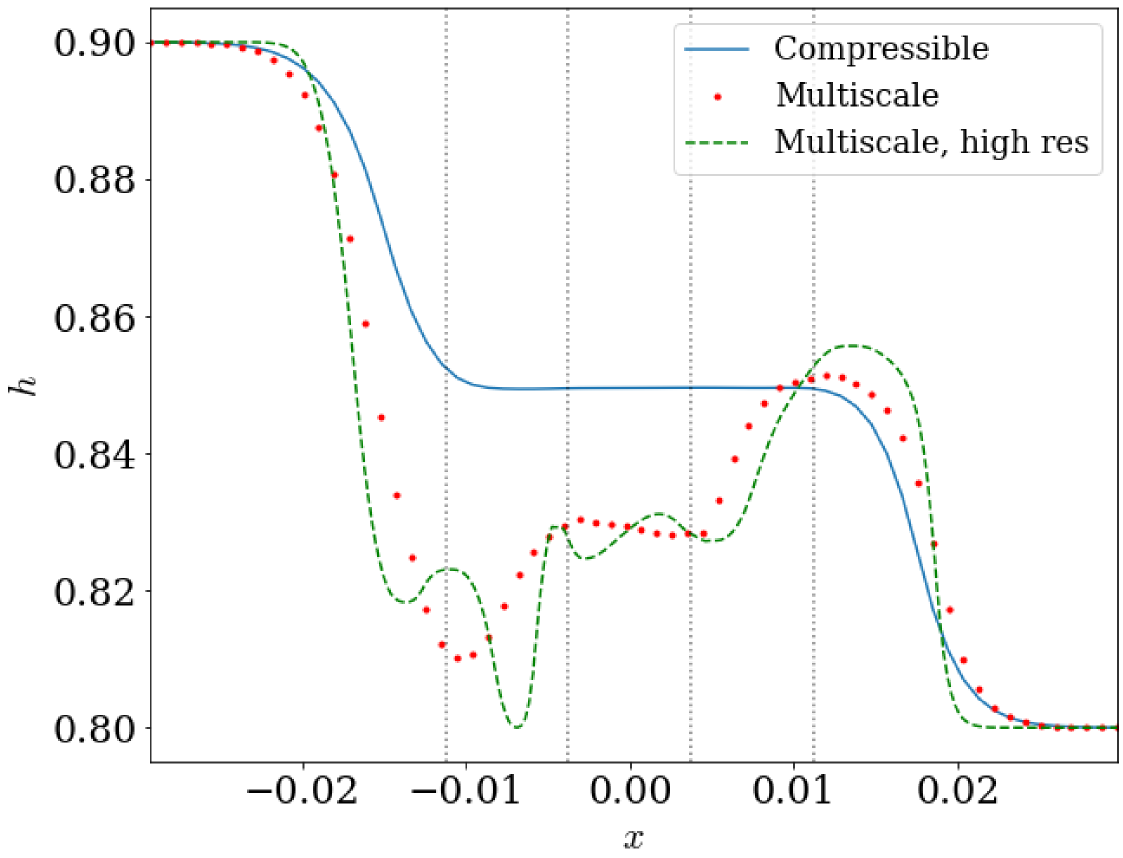
- **AMR** models system on set of nested grids of increasing resolution
- **AMAR** (*adaptive mesh and algorithm refinement*) uses different physical models or numerical schemes on different levels

Multiscale model

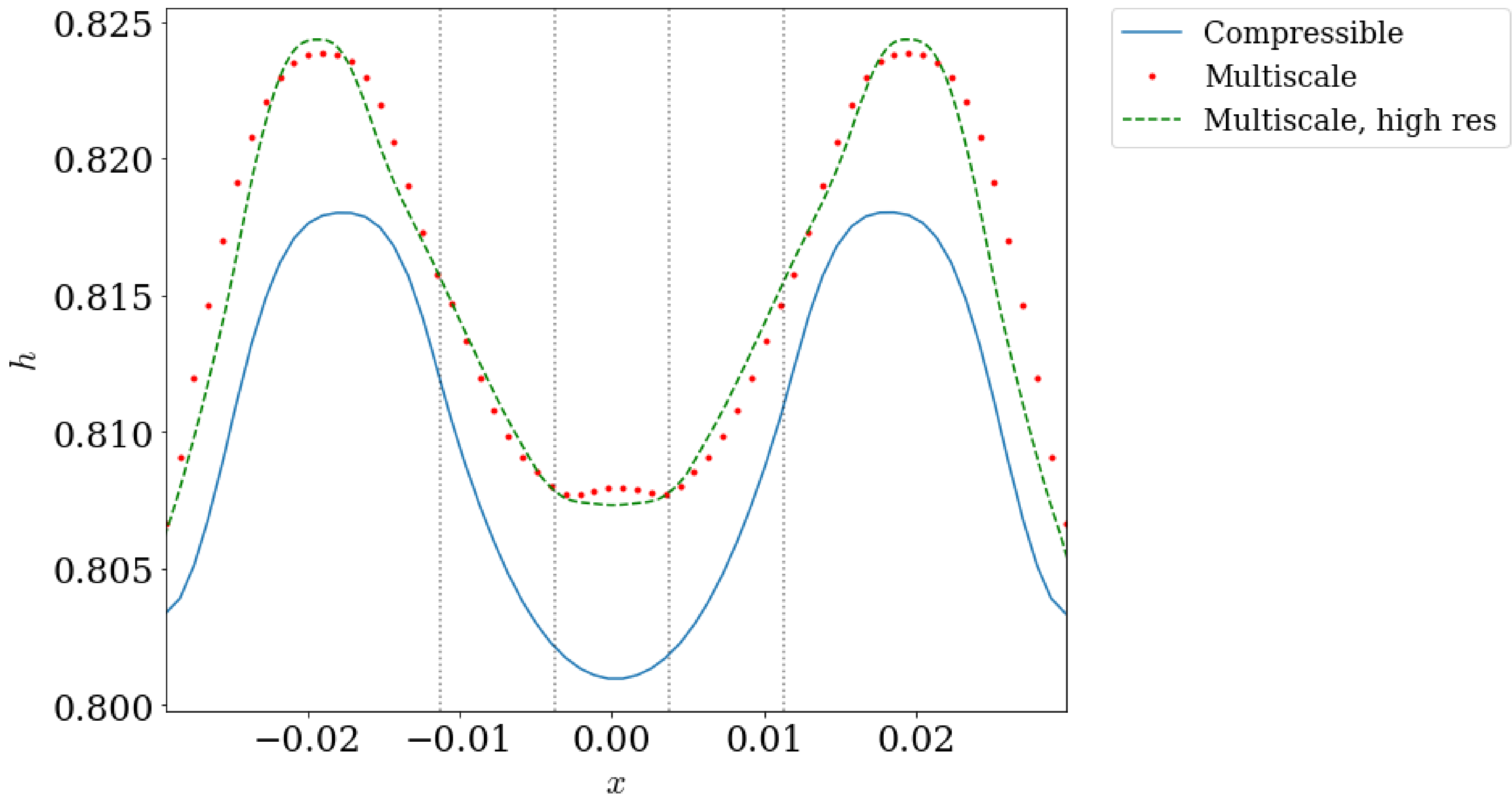
- Goal: SW on coarsest grids, compressible on intermediate, low Mach on finest
- So far concentrated on large scale - **SW & compressible**
- Built using `Castro` (Almgren+ 2010) & `AMReX` (Zhang+ 2016) codes
- Motheau+ 18 have been working on complementary model with LM & compressible

Multiscale challenges

- **Converting consistently** between the different sets of conserved variables at model interfaces
  - At interface, assume **adiabaticity** and **HSE**
- SW to compressible: convert between 2d & 3d models
  - **Integrate vertically** (syncing across patches where necessary)
- Current implementation does ok with small, smooth features, but serious(!) issues with sharper features
  - Above assumptions not valid for this system?
  - Could use e.g. Ghost Fluid Method to help match models



Multiscale dam break w/ 3 levels of (fixed) mesh refinement: 1 coarse SW level, 2 finer compressible levels centred about $x = 0$ (see dotted grey lines)



Multiscale Gaussian w/ 3 levels of (fixed) mesh refinement: 1 coarse SW level, 2 finer compressible levels centred about $x = 0$ (see dotted grey lines)

Future plans

- **Improve matching** of models at interface
- Add **low Mach** to model turbulent burning
- Extend relativistic SW to general spacetime
- Do some physics! Investigate effects of rotation & strong (GR) gravity

Conclusions

- We are looking at effects of modelling **strong gravity** in bursts using GR
- Using a **multiscale model** of relativistic shallow water equations & compressible hydro
- Model still needs some work improving matching of models at interface
- Next steps: add LM, model real physics

Additional slides

Relativistic fluid equations & Wilson formulation

- General relativistic fluid equations:

$$\nabla_\mu (\rho u^\mu) = 0$$

$$u^\mu \nabla_\mu (\rho h - p) + \rho h \nabla_\mu u^\mu = 0$$

$$\rho h u^\nu \partial_\nu u_\mu + \partial_\mu p + u_\mu u^\nu \partial_\nu p = \rho h \Gamma_{\rho\nu\mu} u^\nu u^\rho$$

- Wilson formulation :

$$D = \rho u^0 \quad \text{and} \quad U^\mu = u^\mu / u^0$$

GR Low Mach number equations

- Continuity
  $\partial_t D + \partial_i \left(D U^i \right) = -D \Gamma^{\mu}_{\mu\nu} U^{\nu}$
- Energy
  $\partial_t \left(Dh \right) + \partial_i \left(U^i Dh \right) = u^0 \frac{Dp_0}{Dt} - Dh \Gamma^{\mu}_{\mu\nu} U^{\nu}$
- Momentum
  $\partial_t U_j + U^i \partial_i U_j = -U_j \frac{D \ln u^0}{Dt} + \Gamma_{\rho\nu j} U^{\nu} U^{\rho} - \frac{1}{Dhu^0} \left(\partial_j p_0 + \xi \partial_j \left[\frac{\pi}{\xi} \right] \right)$
- Velocity constraint
  $\partial_i \left(\zeta U^i \right) = \zeta \left(S - \frac{1}{\Gamma_1 p_0} \frac{\partial p_0}{\partial t} \right)$
- Integrating factor
  $\zeta\left(r, t \right) = \zeta\left(0, t \right) \exp \left[\int_0^r dr' \, \frac{1}{\Gamma_1 p_0} \frac{\partial p_0}{\partial r'} \right]$